\[
\sigma_m = \frac{-4369.7 + \sqrt{4369.7^2 + 4(3.4577)(10^2)}}{2} = 684.2 \text{ MPa}
\]

\[
\sigma_a = 684.2 - 152.3 = 531.9 \text{ MPa}
\]

\[
n_f = \frac{531.9}{152.3} = 3.49
\]

Thus, the spring is not likely to fail in fatigue at the outer radius.  

7-22  The solution at the inner radius is the same as in Prob. 7-21. At the outer radius, the yield solution is the same.

Fatigue line:

\[
\sigma_a = \left(1 - \frac{\sigma_m}{S_{ut}}\right)S_e = \sigma_m - 152.3
\]

\[
639 \left(1 - \frac{\sigma_m}{1671}\right) = \sigma_m - 152.3
\]

\[
1.382\sigma_m = 791.3 \quad \Rightarrow \quad \sigma_m = 572.4 \text{ MPa}
\]

\[
\sigma_a = 572.4 - 152.3 = 420 \text{ MPa}
\]

\[
n_f = \frac{420}{152.3} = 2.76 \quad \text{Ans.}
\]

7-23  Preliminaries:

Table A-20:  
\[S_{ut} = 64 \text{ kpsi}, \quad S_f = 54 \text{ kpsi}\]

\[S'_e = 0.504(64) = 32.3 \text{ kpsi}\]

\[k_a = 2.70(64)^{-0.265} = 0.897\]

\[k_b = 1\]

\[k_c = 0.85\]

\[S_e = 0.897(1)(0.85)(32.3) = 24.6 \text{ kpsi}\]

\text{Fillet:}

Fig. A-15-5:  
\[D = 3.75 \text{ in}, \quad d = 2.5 \text{ in}, \quad D/d = 3.75/2.5 = 1.5, \text{ and } r/d = 0.25/2.5 = 0.10\]

\[. \quad \therefore K_f = 2.1\]

\[
K_f = \frac{2.1}{1 + (2/\sqrt{0.25})[(2.1 - 1)/2.1](4/64)} = 1.86
\]

\[
\sigma_{\text{max}} = \frac{4}{2.5(0.5)} = 3.2 \text{ kpsi}
\]

\[
\sigma_{\text{min}} = \frac{-16}{2.5(0.5)} = -12.8 \text{ kpsi}
\]

\[
\sigma_a = 1.86 \left|\frac{3.2 - (-12.8)}{2}\right| = 14.88 \text{ kpsi}
\]
\[ \sigma_m = 1.86 \left[ \frac{3.2 + (-12.8)}{2} \right] = -8.93 \text{ kpsi} \]

\[ n_y = \left| \frac{S_y}{\sigma_{\text{min}}} \right| = \left| \frac{54}{-12.8} \right| = 4.22 \]

Since the midrange stress is negative,

\[ S_a = S_e = 24.6 \text{ kpsi} \]

\[ n_f = \frac{S_a}{\sigma_a} = \frac{24.6}{14.88} = 1.65 \]

**Hole:**

Fig. A-15-1: \( d/w = 0.75/3.75 = 0.20 \), \( K_f = 2.5 \)

\[ K_f = \frac{2.5}{1 + (2/\sqrt{0.75/2})(2.5 - 1)/2.5(5/64)} = 2.17 \]

\[ \sigma_{\text{max}} = \frac{4}{0.5(3.75 - 0.75)} = 2.67 \text{ kpsi} \]

\[ \sigma_{\text{min}} = \frac{-16}{0.5(3.75 - 0.75)} = -10.67 \text{ kpsi} \]

\[ \sigma_a = 2.17 \left| \frac{2.67 - (-10.67)}{2} \right| = 14.47 \text{ kpsi} \]

\[ \sigma_m = 2.17 \left| \frac{2.67 + (-10.67)}{2} \right| = -8.68 \text{ kpsi} \]

Since the midrange stress is negative,

\[ n_y = \left| \frac{S_y}{\sigma_{\text{min}}} \right| = \left| \frac{54}{-10.67} \right| = 5.06 \]

\[ S_a = S_e = 24.6 \text{ kpsi} \]

\[ n_f = \frac{S_a}{\sigma_a} = \frac{24.6}{14.47} = 1.70 \]

Thus the design is controlled by the threat of fatigue at the fillet; the minimum factor of safety is \( n_f = 1.65 \). **Ans.**

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7.24 (a) \( M = -T \), \( h = 5 \text{ mm} \), \( A = 25 \text{ mm}^2 \)

\[ r_c = 20 \text{ mm}, \quad r_o = 22.5 \text{ mm}, \quad r_i = 17.5 \text{ mm} \]

\[ r_n = \frac{h}{\ln r_o/r_i} = \frac{5}{\ln (22.5/17.5)} = 19.8954 \text{ mm} \]

\[ e = r_c - r_n = 20 - 19.8954 = 0.1046 \text{ mm} \]

\[ c_o = 2.605 \text{ mm}, \quad c_i = 2.395 \text{ mm} \]