Text problems:

7.26
(a) You can use the Modified Goodman relationship from class and substitute shear stresses ($\tau_m, \tau_a$) for the normal stresses ($\sigma_m, \sigma_a$). Make sure that you use the ultimate shear strength and the shear yield strength, which can be computed as follows:

$$S_{us} \approx \frac{2}{3} S_{ut} \quad (1)$$

$$S_{ys} \approx \frac{1}{\sqrt{3}} S_y \quad (2)$$

(b) Remember that since the shaft is not rotating, you need to use the effective diameter instead of the actual diameter for computing $k_b$.

(c) When computing $S'_e$ from the ultimate strength always use the first equation that I gave you (equation 7-8 in the book), because he load factor ($k_c$) accounts for the different types of loading.

(d) The answers are (a) $n \approx 1.04$ (b) $n \approx 1.7$

7.27
(a) The book says to make sure that $1 \times 10^6$ cycles can be withstood, but do the design for infinite life.

(b) When you have combined loads in fatigue, you can substitute the von Mises equivalent mean and amplitude stresses into the Modified Goodman criteria.

(c) The answers are (a) $P = 15.8 \, kN$; $n_y = 5.7$ (b) $P = 51.4 \, kN$; $n_y = 3.87$

Non-text problems:

1. Design the diameter of a circular shaft that is fixed to a wall at one end and has a torque of 10 kip-in applied to the other for a reliability of 99.7%. The mean yield strength of the material is $S_y = 50$ ksi, and the standard deviation is $\sigma_S = 5$ ksi. The torque has a standard deviation of $\sigma_L = 2$ kip-in. (Answer: $d = 1.21$ in).

Bonus: (50 pts) You are designing a circular beam, pictured in figure 1, that is subjected to end moments of varying magnitude. Given the following design requirements, determine what the diameter of the beam should be for a factor of safety $n$, using the values in table 1:

(a) $1 \times 10^6$ cycles varying between zero and a moment, $M$
(b) $1 \times 10^5$ cycles varying between zero and a moment, $9/8*M$
(c) $1 \times 10^4$ cycles varying between zero and a moment, $10/8*M$
(d) $1 \times 10^3$ cycles varying between zero and a moment, $11/8*M$
(e) A reliability of 95% is required

The S-N diagram for bending of the design material is shown in figure 2. You may make the following assumptions:
Table 1: Data values for non-text problem

<table>
<thead>
<tr>
<th>M</th>
<th>$S_{ut}$</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 kip-in</td>
<td>100 ksi</td>
<td>2</td>
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Figure 1: Beam diagram non-text problem

(a) Assume that the relationship between number of cycles and $S0N$ is linear when $N$ is in log space. This allows an equation to be developed that relates $S0N$ and $N$, so that the design may be done automatically, rather than always reading numbers off a chart. Based on figure 2, the following relationships can be derived:

$$S'_f = -10 \log N + 120 \quad 1 \times 10^3 \leq N \leq 1 \times 10^7$$  
$$S'_f = 50 \quad 1 \times 10^7 < N$$

The factor of safety will take care of the fact that the linear assumption is too aggressive.

(b) Assume that computing the modified endurance strength ($S'_f$) from $S'_f$ can be done in the same manner as for the modified endurance limit ($S_e$). Assume that all factors will remain the same.

(c) Assume that the Goodman relationship is valid for the mean and alternating stresses and the Palmgren-Miner rule accurately describes the cumulative damage.

***The use of Maple or Mathcad is strongly recommended for this problem. You will not be able to solve for the required diameter without using a numerical solver that the programs have, or a graphical solution. Please include all plots, and computer work in the homework. Do not use your graphing calculator, since you cannot print out your plots, etc.***