Chapter 3

The Cellular Concept
– System Design Fundamentals

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Outlines

Frequency Reusing
Design of Clutter Size
Channel Assignment Strategies
Handoff Strategies
Co-channel Interference and Adjacent channel Interference
Trunking and Grade of Service
Early Mobile Telephone System

Traditional mobile service was similar to radio/TV broadcasting: One very powerful transmitter located at the highest spot in a large area.

Cellular System

In a cellular system, instead of using one powerful transmitter, many low-power transmitters were placed throughout a coverage area.
The cellular concept was a major breakthrough in solving the problem of spectral congestion and user capacity.

- It offered very high capacity in a limited spectrum allocation without any major technological changes.
Cellular System

However, neighboring cells will interfere to each other. Therefore, they shall use different frequency bands. A cell cluster is outlined in bold, and replicated over the coverage area. Cells with the same letter use the same set of frequencies.

In this example, the cluster size is $N=7$, and the frequency reuse factor is $1/7$, since each cell contains $1/7$ of the total number of available channels.
Choices of Hexagonal Cell Geometry

Factors
- Equal area
- No overlap between cells

Choices

For a given \( R \), A3 provides \textbf{maximum coverage area}.

By using hexagon geometry, the fewest number of cells covers a given geographic region.
Frequency Reusing

Actual cellular footprint is determined by the contour of a given transmitting antenna.
Frequency Reusing

Consider a cellular system

Total duplex channels : $S$
Cluster size : $N$ cells
No. of channels in each cell : $k = \frac{S}{N}$
Capacity in a cluster : $C = kN = S$

If a cluster is replicated $M$ times
Total capacity : $C = MkN = MS$

→ The capacity is increased by $M$. 
Design of Cluster Size

**To Find the Nearest Co-channel Neighbor of Particular Cell:**

Move $i$ cells along any chain or hexagon. Then turn 60 degrees counterclockwise and move $j$ cells.

In this example, $i=3$, $j=2$, $N=19$. 
Design of Cluster Size

In order to connect without gaps between adjacent cells

\[ N = i^2 + ij + j^2 \]

where \( i \) and \( j \) are non-negative integers.

Example: When \( i = 2, j = 1 \)

\[ N = 2^2 + 2(1) + 1^2 = 4 + 2 + 1 = 7 \]

Commonly used cluster size \( N = 4, 7, 12 \).
Example

Problem:
If a particular FDD cellular telephone system has a total bandwidth of 33 MHz,
and if the phone system uses two 25 KHz simplex channels to provide full duplex voice and control channels...
compute the number of channels per cell if $N = 4, 7, 12$.

Solution:
Total bandwidth = 33 MHz
Channel bandwidth = 25 KHz x 2 = 50 KHz
Total available channels = 33 MHz / 50 KHz = 660

$N = 4$ Channel per cell = 660 / 4 = 165 channels
$N = 7$ Channel per cell = 660 / 7 = 95 channels
$N = 12$ Channel per cell = 660 / 12 = 55 channels
Channel Assignment Strategies

**Fixed Channel Assignment Strategy:**

Each cell is allocated a predetermined set of voice channels.

If all the channels in that cell are occupied, the call is blocked, and the subscriber does not receive service.

*Variation includes a borrowing strategy:*

- A cell is allowed to borrow channels from a neighboring cell if all its own channels are occupied.
- This is supervised by the mobile switch center (MSC).

**Dynamic Channel Assignment Strategy:**

Voice channels are not allocated to different cells permanently.

Each time a call request is made, the serving base station requests a channel from the MSC.

The switch then allocates a channel to the requested call, based on a decision algorithm taking into account different factors - frequency re-use of candidate channel, cost factors.

Dynamic channel assignment is more complex (real time), but reduces likelihood of blocking.
Handoff Strategies

Handoff:
When a mobile moves into a different cell while a conversation is in progress, the MSC automatically transfers the call to a new channel belonging to the new base station.

- Important task in any cellular radio system
- Handoffs must be performed successfully, as infrequently as possible, and not visible to users.

When designing handoff, it is important to bear in mind that it requires certain time to complete the handoff.
Handoff Strategies

(a) Improper Handoff Situation

\[ \text{Margin} = P_{r, \text{handoff}} - P_{r, \text{min usable}} \]
Handoff Strategies

Margin $? = P_{r, \text{handoff}} - P_{r, \text{min usable}}$ should be optimal value.

If $?$ is too large: too many handoffs
If $?$ is too small: chance of call being lost

Note that there is a time delay to complete the handoff.

Excessive delay at the MSC may occur during high traffic conditions
- due to computational loading at the MSC
- no channels are available on any of the nearby base stations
Handoff Strategies

(b) Proper Handoff Situation

- Received signal level
- Time
- Level at point B
- Level at which handoff is made
- Call properly transferred to BS2

BS1

BS2

A

B
In deciding when to handoff, it is important to ensure that the drop in the measured signal level is not due to momentary fading.

Each base station constantly monitors the signal strength of all its reverse voice channels to determine the relative location of each mobile user with respect to the base station tower.

**Dwell Time:** time over which a call may be maintained within a cell, without hand-off.

Dwell time of a particular user is governed by a number of factors, including propagation, interference, distance between the subscriber and the base station, and other time-varying effects.
Handoff Strategies

Mobile assisted hand-off (MAHO) : Every mobile station measures the received power from surrounding base stations and continuously reports the results of these measurements to the serving base station. → Faster hand-off rate.

Inter-system handoff : One cellular system to a different cellular system.

Prioritizing Handoffs: From the user’s point of view, having a call abruptly terminated while in the middle of a conversation is more annoying than being blocked occasionally on a new call attempt.

One method for giving priority to handoffs is called the guard channel concept, whereby a fraction of the total available channels in a cell is reserved exclusively for handoff requests from ongoing calls which may be handed off into the cell.
Interference

Major limiting factor in performance of cellular radio systems:

• **Co-channel interference**
  - Cells that use the same set of frequencies are called co-channel cells.
  - Interference between them is called co-channel interference.

• **Adjacent channel interference**
  - Interference resulting from signals which are adjacent in frequency to the desired signal.
  - Due to imperfect receiver filters that allow nearby frequencies to leak into pass band.
Co-Channel Interference

Signal to interference ratio (SIR, or $S/I$) for a mobile receiver is given by:

$$\frac{S}{I} = \text{SIR} = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

$S = \text{desired signal power from designated base station}$

$I_i = \text{interference power caused by the } i\text{th interfering co-channel cell base station}$

$i_0 = \text{number of interfering co-channel cells}$
Co-channel Interference

Co-channel reuse ratio:

\[ Q = \frac{D}{R} \]

\( R \) = radius of the cell

\( D \) = distance between centers of nearest co-channel cells

For hexagonal geometry,

\[ Q = \frac{D}{R} = \sqrt{3N} \]
Co-Channel Interference

For any given antenna (base station) the power at a distance $d$ is given by:

$$P_r(d) = P_r(d_0) \left( \frac{d}{d_0} \right)^{-n}$$

or

$$P_r(d) \text{ (dBm)} = P_r(d_0) \text{ (dBm)} - 10n \log \left( \frac{d}{d_0} \right)$$

$P_r(d_0)$ : power receiver at a close-in reference point in the far field region of the antenna

$d_0$ : distance from the transmitting antenna to the reference point

$n = \text{path loss exponent (between 2-4 in urban cellular systems)}$
Co-Channel Interference

When different base stations transmit the same power, the SIR becomes

\[
\frac{S}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_0} (D_i)^{-n}}
\]

\(i_0\) = number of first tier interfering co-channel cells

\(R\) = radius of the cell

\(D_i\) = distance of the \(i\)th interferer from the mobile
Co-Channel Interference

Equidistant case \((D_i = D)\)

\[
\frac{S}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_0} (D)^{-n}} = \frac{(D / R)^{n}}{i_0} = \frac{\left(\sqrt{3N}\right)^n}{i_0}
\]

To achieve 18 dB SIR (required in US AMPS), \(N > 6.49\) provided \(n = 4\).

That is, \(N \geq 7\) should be used.
**Co-Channel Interference**

**Worst case**: when the mobile is at the cell boundary (point x). The marked distances are based on approximations made for easy analysis.

\[
\frac{S}{I} = \frac{R^{-4}}{2(D-R)^{-4} + 2(D+R)^{-4} + 2D^{-4}}
\]

\[
= \frac{1}{2(Q-1)^{-4} + 2(Q+1)^{-4} + 2Q^{-4}}
\]

When \(N = 7\), \(Q = 4.6\)

- SIR=17 dB (using above approx.)
- SIR=17.8 using exact expression

Because the worst case rarely occurs, \(N = 7\) is usually acceptable.
Adjacent Channel Interference

- Interference resulting from signals which are adjacent in frequency to the desired signal.
- Due to imperfect receiver filters that allow nearby frequencies to leak into pass band.
- Performance may degrade seriously due to *near-far* effect.
- Can be minimized by careful filtering and assignments; and, by keeping frequency separation between channel in a given cell as large as possible.
- A channel separation greater than six is needed to bring the adjacent channel interference to an acceptable level.
Trunking and Grade of Service

- Cellular radio systems rely on trunking to accommodate a large number of users in a limited radio spectrum. The concept of trunking allows a large population to be accommodated by a limited number of services.

- **Trunking**: each user is allocated a channel on a per-call basis; and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

  Trunking theory was initiated by Danish mathematician, A. K. Erlang.

- **Measure of traffic intensity**: one Erlang represents the amount of traffic intensity carried by a channel that is completely occupied. For example, a radio channel that is occupied for 30 minutes during an hour carries 0.5 Erlang of traffic.

- **Grade of Service (GOS)**: Measure of ability of the user to access a trunked system during the busiest hour during a week, month or year.

  GOS is typically given as the likelihood that a call is blocked, or the likelihood of a call experiencing a delay greater than a certain queuing time.
Some Definitions

**Table 3.3** Definitions of Common Terms Used in Trunking Theory

*Set-up Time:* The time required to allocate a trunked radio channel to a requesting user.

*Blocked Call:* Call which cannot be completed at time of request, due to congestion. Also referred to as a *lost call.*

*Holding Time:* Average duration of a typical call. Denoted by $H$ (in seconds).

*Traffic Intensity:* Measure of channel time utilization, which is the average channel occupancy measured in Erlangs. This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels. Denoted by $A$.

*Load:* Traffic intensity across the entire trunked radio system, measured in Erlangs.

*Grade of Service (GOS):* A measure of congestion which is specified as the probability of a call being blocked (for Erlang B), or the probability of a call being delayed beyond a certain amount of time (for Erlang C).

*Request Rate:* The average number of call requests per unit time. Denoted by $\lambda$ seconds$^{-1}$. 
Traffic intensity of each user:

\[ A_\mu = \lambda H \] (Erlang)

\[ \lambda : \text{Average number of call requests per unit time} \]
\[ H : \text{duration of a call} \]

For system entering \( U \) users, the total offered traffic intensity

\[ A = U A_\mu \] (Erlangs)

If there are \( C \) channels in the system, average intensity per channel is

\[ A_c = A / C = U A_\mu / C \]
Trunking and Grade of Service

There are two types of trunked systems:

> **Blocked Calls Cleared System**

  - No queuing for call requests
  - If no channels are available, the requesting user is blocked without access and is free to try again later.

> **Blocked Calls Delayed System**

  - Queue is provided to hold calls which are blocked. If a channel is not available immediately, the call request may be delayed until a channel becomes available.
Calculation of GOS – Erlang B System

Blocked Calls Cleared Formula (Erlang B Formula)

Assuming
(a) There are an infinite number of users
(b) There are memoryless arrivals of requests (i.e., all users, including the blocked users, may request a channel at any time)
(c) The probability of a user occupying a channel is exponentially distributed (long calls are less likely to occur)
(d) There are a finite number of available channels \( C \)

Then (proof is given in Appendix A of the textbook)

\[
GOS = \Pr \text{ [call is blocked]} = \frac{\sum_{k=0}^{C} \frac{A^k}{k!}}{C!} \quad \text{(Erlang B Formula)}
\]
### Calculation of GOS – Erlang B System

#### Table 3.4  Capacity of an Erlang B System

<table>
<thead>
<tr>
<th>Number of Channels $C$</th>
<th>Capacity (Erlangs) for GOS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$= 0.01$</td>
<td>$= 0.005$</td>
<td>$= 0.002$</td>
<td>$= 0.001$</td>
</tr>
<tr>
<td>2</td>
<td>0.153</td>
<td>0.105</td>
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<td>0.869</td>
<td>0.701</td>
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<td>1.13</td>
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<td>100</td>
<td>84.1</td>
<td>80.9</td>
<td>77.4</td>
<td>75.2</td>
</tr>
</tbody>
</table>
(Erlang B System)

A free online calculator is available at http://www.erlang.com/calculator/erlb/
Calculation of GOS – Erlang C System

Blocked Calls Delayed Formula (Erlang C Formula)

The likelihood of a call not having immediate access to a channel

$$\Pr [\text{delay} > 0] = \frac{A^C}{A^C + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{C-1} \frac{A^k}{k!}}$$

(Erlang C Formula)

The probability that the delay is greater than \( t \) is

$$\Pr[\text{delay} > t] = \Pr[\text{delay} > 0] \Pr[\text{delay} > t | \text{delay} > 0]$$

$$= \Pr[\text{delay} > 0] \exp\left(-\frac{(C - A)t}{H}\right)$$

\( H \): holding time (in sec), i.e., average duration of a typical call.

The averaged delay \( D \) for all calls is

$$D = \Pr[\text{delay} > 0] \frac{H}{C - A}$$

The averaged delay for calls which are queued is

$$\frac{H}{C - A}$$
A free online calculator is available at http://www.erlang.com/calculator/erlc/
Problem:
- A hexagonal cell in a 4-cell system has a radius of 1.387km; and a total of 60 channels are used within the entire system.
- If the load / user is 0.029 Erlangs, \( \lambda = 1 \) call per hour, compute the following for an Erlang C system that has a 5% probability of a delayed call.
  a. How many users per square km will the system support?
  b. What is the Pr [ Delay > 10s ]?

Solution:
- Cell radius \( R = 1.387 \) km
- The area of a hexagonal cell is \( 2.5981R^2 \).
  Therefore, area covered per cell = \( 2.598 \times (1.387)^2 \) = 5 sq km
- Number of cells per cluster = 4
- Total number of channels per cell = \( 60 / 4 = 15 \) channels
Solution (con’t):

(a)

GOS = 0.05, \( C = 15 \),

- From Erlang C chart, traffic intensity \( A = 9.0 \) Erlangs
- Number of users
  = total traffic intensity / Traffic per user = 9.0 / 0.029 = 310 users
- Number of users per sq. km = 310 / 5 = 62 users per sq. km.

(b)

\[
H = A_\mu / \lambda = 0.029 \text{ hr} = 0.029 \times 60 \times 60 \text{ seconds} = 104.4 \text{ seconds}
\]

\[
\Pr [\text{Delay} > 10\text{s}] = \Pr [\text{Delay} > 0 ] e^{-(C-A) t / H} = 0.05 e^{-(15-9) \times 10 / 104.4} = 0.0281 = 2.81\%
\]
Improving Capacity in Cellular Systems

- As demand for wireless services increases, the number of channels assigned to a cell is not enough to support the required number of users.
- Solution is to increase channels per unit coverage area.

**Cell Splitting**:
- Subdivides a congested cell into smaller cells, each with its own base station.
- With $R$ decreased and $D/R$ unchanged, the capacity of a cellular system is increased.
Improving Capacity in Cellular Systems

Cell Splitting:

Figure 3.9  Illustration of cell splitting within a 3 km by 3 km square centered around base station A.
Improving Capacity in Cellular Systems

Sectoring:

- **Sectoring:** The technique for decreasing co-channel interference by using directional antennas.

- A single omni-directional antenna at the base station is replaced by several directional antennas, each radiating within a specified sector.

- A given cell will receive interference and transmit with only a fraction of the available co-channel cells.

(a) 120° sectoring

(b) 60° sectoring
Improving Capacity in Cellular Systems

Sectoring
Improving Capacity in Cellular Systems

Sectoring
Sectoring (con’t):

- In this example (seven-cell reuse, 120° sectors), the number of interferers in the first tier is reduced from 6 to 2.
- SIR is improved from 17 dB to 24.2 dB.
- Additional SIR improvement is possible by down-tilting the sector antennas.

Figure 3.11 Illustration of how 120° sectoring reduces interference from co-channel cells. Out of the 6 co-channel cells in the first tier, only two of them interfere with the center cell. If omnidirectional antennas were used at each base station, all six co-channel cells would interfere with the center cell.
Improving Capacity in Cellular Systems

Micro Cell Zone Concept:

- Large control base station is replaced by several lower powered transmitters on the edge of the cell.
- The mobile retains the same channel and the base station simply switches the channel to a different zone site and the mobile moves from zone to zone.
- Since a given channel is active only in a particular zone in which mobile is traveling, base station radiation is localized and interference is reduced.
- The channels are distributed in time and space by all three zones are reused in co-channel cells.
- Advantage is that while the cell maintains a particular coverage radius, co-channel interference is reduced due to zone transmitters on edge of the cell.
Improving Capacity in Cellular Systems

Micro Cell Zone Concept:

[Diagram showing Micro Cell Zone Concept with Tx/Rx, Zone Selector, Base Station, and Microwave or fiber optic link connections.]
Example 3.4
How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

Solution
From Table 3.4, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels. By using the relation \( A = UA_u \), we can obtain the total number of users that can be supported in the system.

(b) Given \( C = 5 \), \( A_u = 0.1 \), \( GOS = 0.005 \)
From Figure 3.6, we obtain \( A = 1.13 \).
Therefore, total number of users, \( U = A/A_u = 1.13/0.1 \approx 11 \) users.

(e) Given \( C = 100 \), \( A_u = 0.1 \), \( GOS = 0.005 \),
From Figure 3.6, we obtain \( A = 80.9 \).
Therefore, total number of users, \( U = A/A_u = 80.9/0.1 = 809 \) users.
Example 3.5
An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution
System A
Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, C = 19
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 19$, from the Erlang B chart, the total carried traffic, $A$, is obtained as 12 Erlangs. Therefore, the number of users that can be supported per cell is $U = A/A_u = 12/0.1 = 120$

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to $120 \times 394 = 47280$
System B
Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 57$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total
carried traffic, $A$, is obtained as 45 Erlangs.
Therefore, the number of users that can be supported per cell is
$U = A/A_u = 45/0.1 = 450$
Since there are 98 cells, the total number of subscribers that can
be supported by System B is equal to $450 \times 98 = 44,100$

System C
Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 100$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total
carried traffic, $A$, is obtained as 88 Erlangs.
Therefore, the number of users that can be supported per cell is
$U = A/A_u = 88/0.1 = 880$
Since there are 49 cells, the total number of subscribers that can be
supported by System C is equal to $880 \times 49 = 43,120$
Therefore, total number of cellular subscribers that can be supported by these three systems are $47,280 + 44,100 + 43,120 = 134,500$ users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to $47,280/2,000,000 = 2.36\%$.

Similarly, market penetration of System B is equal to $44,100/2,000,000 = 2.205\%$

and the market penetration of System C is equal to $43,120/2,000,000 = 2.156\%$

The market penetration of the three systems combined is equal to $134,500/2,000,000 = 6.725\%$.
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