Chapter 4
Mobile Radio Propagation:
Large-Scale Path Loss

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Outlines

Propagation Model - Attenuations
  - Free Space
  - Reflection
  - Diffraction
  - Scattering

Log-Normal Shadowing

Practical Link Budget Design
Propagation Models

![Graph showing received power vs T-R separation (meters)]
Mobile Radio Propagation

- Mobile radio channel is an important controlling factor in wireless communication systems.
- Transmission path between transmitter and receiver can vary in complexity.
- Wired channels are stationary and predictable, whereas radio channels are extremely random and have complex models.
- Modeling of radio channels is done in statistical fashion based on measurements for each individual communication system or frequency spectrum.
Propagation Models

- **Large scale propagation models** - To predict the average received signal strength over large T-R separation distances (several hundreds or thousands of meters).
  
  Typically, the local average received power is computed by averaging signal measurements over a measurement track of 5 to 40 wavelengths.

- **Small scale propagation models (or fading models)** - To characterize the rapid fluctuations of the receiver signal strength over very short distances (a few wavelengths) or short time durations (on the order of seconds).

  In small scale fading, the received signal power may vary by as much as three to four orders of magnitude (30 to 40 dB).
Electrical Field

The electric field is expressed as a vector \( \mathbf{E} \)

\[
\mathbf{E} = \vec{x} E_x + \vec{y} E_y + \vec{z} E_z
\]

and its magnitude is given by

\[
E = |\mathbf{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}
\]

The unit of electrical field is volts/meter.
The standard unit of power is Watt, but dBm is more commonly used.

\[ P \text{ (dBm)} = 10 \log_{10} [ P \text{ (mW)}] \]

<table>
<thead>
<tr>
<th>( P \text{ (mW)} )</th>
<th>( P \text{ (dBm)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( 10^{-1} )</td>
<td>-10</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>-20</td>
</tr>
</tbody>
</table>

Minimum usable signal strength to be received at a base station is typically between -90 dBm -100 dBm.
Propagation Models

- **Free Space Propagation**: Transmitter and receiver have a clear, unobstructed LOS path between them.
- **Reflection**: From the surface of the earth and from buildings and walls. Usually dimensions of reflecting object are much greater than wavelength.
- **Diffraction**: Bending of electromagnetic waves around sharp edges such as, sharp towers or peaks.
- **Scattering**: Due to objects in the medium that are small compared to wavelength and the number of objects is many (e.g., foliage, street signs, lamp posts, rain, shower).
Figure 4.2 Illustration of a linear radiator of length $L (L \ll \lambda)$, carrying a current of amplitude $i_0$ and making an angle $\theta$ with a point, at distance $d'$. 
Radiating Power to Electric Field

Electric and magnetic fields launched from electric current $i_0$

$$E_r = \frac{i_0 L \cos \theta}{2\pi \varepsilon_0 c} \left[ \frac{1}{d^2} + \frac{c}{j \omega_c d^3} \right] e^{j \omega_c (t - d / c)}$$

$$E_\theta = \frac{i_0 L \sin \theta}{4\pi \varepsilon_0 c^2} \left[ \frac{j \omega_c}{d} + \frac{c}{d^2} + \frac{c^2}{j \omega_c d^3} \right] e^{-j \omega_c (t - d / c)}$$

$$H_\phi = \frac{i_0 L \sin \theta}{4\pi c} \left[ \frac{j \omega_c}{d} + \frac{c}{d^2} \right] e^{j \omega_c (t - d / c)}$$

$$E_\phi = H_r = H_\theta = 0$$

In the near field ($d$ is small), $1/d^2$ and $1/d^3$ terms may dominate.
Radiating Power to Electric Field

When $d$ is large, $1/d^2$ and $1/d^3$ terms becomes negligible, and only

$$E_\theta = \frac{i_0 L \sin \theta}{4\pi \varepsilon_0 c^2} \left[ \frac{j \omega_c}{d} \right] e^{-j\omega_c (t-d/c)}$$

$$H_\phi = \frac{i_0 L \sin \theta}{4\pi c} \left[ \frac{j \omega_c}{d} \right] e^{j\omega_c (t-d/c)}$$

survive.

**Far field regions**: regions far away from the transmitter satisfying

$$d >> d_f = 2D^2/\lambda \quad (d_f \text{ is called the Fraunhofer distance})$$

Additionally, to be in the far-field region, $d_f$ must also satisfy

$$d_f >> \lambda \quad \text{and} \quad d_f >> D$$

Hereafter, we only consider propagation at far-field regions.
Effective isotropic radiated power (EIRP)

\[
\text{EIRP} = P_t G_t
\]

where \( P_t \): Transmitter power
\( G_t \): Transmitter antenna gain

EIRP represents the maximum radiated power available from a transmitter in the direction of maximum antenna gain, compared to an isotropic radiator.

In practice, **effective radiated power (ERP)** is more commonly used

\[
\text{ERP} = \text{EIRP} / 1.64 \quad \text{or} \quad \text{ERP (dB)} = \text{EIRP (dB)} - 2.15
\]

ERP represents the maximum radiated power available from a transmitter in the direction of maximum antenna gain, compared to a half-wave dipole antenna.
Free Space Propagation

Power flux density:

\[ P_d = \frac{\text{EIRP}}{4\pi d^2} = \frac{P_t G_t}{4\pi d^2} = \frac{E^2}{\eta} \quad \text{(W/m}^2) \]

where \( \eta = 120\pi = 377 \) (\( \Omega \)) is the intrinsic impedance of free space.
Free Space Propagation

It the receiver antenna is modeled as a matched resistive load to the receiver, then the receiver antenna will induce an voltage into the receiver which is half of the open circuit voltage at the antenna.

\[ P_r(d) = \left(\frac{V_{\text{ant}}}{2}\right)^2 = \frac{V_{\text{ant}}^2}{4R_{\text{ant}}} \]
Free Space Propagation

\[ P_r(d) = P_d A_e = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \]

\[ A_e = \frac{G_r \lambda^2}{4\pi} \] : effective aperture (of the receiver antenna)

\( P_r \) : Received power
\( D \) : Max dimension of transmitting antenna
\( G_r \) : Receiver antenna gain
\( L \) : System loss factor (\( L \geq 1 \), transmission lines etc, but not due to propagation)
\( \lambda = \frac{c}{f} = 3 \cdot 10^8 / f \) : Wavelength
  (units – \( f \) : Hz, \( c = 3 \cdot 10^8 \) : meters/sec, \( \lambda \) : meters)
Free Space Propagation

Path loss: signal attenuation as a positive quantity measured in dB, is defined as the difference (in dB) between the effective transmitted power and the received power, and may or may not include the effect of antenna gains.

(when antenna gains are included)

\[ PL(dB) = 10 \log \frac{P_t}{P_r} = -10 \log \left[ \frac{G_t G_r \lambda^2}{(4\pi)^2 d^2} \right] \]

(when antenna gains are excluded)

\[ PL(dB) = 10 \log \frac{P_t}{P_r} = -10 \log \left[ \frac{\lambda^2}{(4\pi)^2 d^2} \right] \]
Free Space Propagation

For free space propagation, the path loss exponent is 2.

\[ P_r(d) = P_r(d_0) \left( \frac{d}{d_0} \right)^{-2} \]

or

\[ P_r(d) \text{ (dBm)} = P_r(d_0) \text{ (dBm)} - 20 \log \left( \frac{d}{d_0} \right) \]

\[ = 10 \log \left[ \frac{P_r(d_0) \text{ (W)}}{0.001 \text{ (W)}} \right] - 20 \log \left( \frac{d}{d_0} \right) \]

For simplicity of computations, the reference distance \( d_0 \) for practical system using low-gain antennas in the 1-2 GHz region is typically chosen to be 1 m in indoor environment and 100 m or 1 km in outdoor environments.
**Example**

**Program:** Given a transmitter produces 50W of power. If this power is applied to a unity gain antenna with 900 MHz carrier frequency, find the received power at a free space distance of 100 m from the antenna.

What is the received power at 10 km? Assume unity gain for the receiver antenna.

**Solution:**

\[ f_c = 900 \text{ MHz} \Rightarrow \lambda = \frac{3 \cdot 10^8}{900 \cdot 10^6} = 0.333 \text{ m}; \]

\[ P_t = 50 \text{ W}; \quad G_t = 1; \quad G_r = 1; \quad L = 1; \]

At \( d = 100 \text{ m} \)

\[
Pr = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} = \frac{50 \times 1 \times 1 \times 0.333^2}{(4\pi)^2 \times 100^2 \times 1} = 3.5 \times 10^{-6} (\text{W}) = 3.5 \times 10^{-3} (\text{mW})
\]

or

\[
Pr (\text{dBm}) = 10 \log(P_r (\text{mW})) = -24.5 \text{ (dBm)}
\]

At \( d = 10 \text{ km} \)

\[
Pr = 3.5 \times 10^{-10} (\text{W}) = 3.5 \times 10^{-7} (\text{mW}) \quad P_r (\text{dBm}) = -64.5 \text{ (dBm)}
\]
Reflection

Electric Properties of Material Bodies

Permittivity \( \varepsilon \)  F/m \( \rightarrow \) Farads/m
Permeability \( \mu \)  H/m \( \rightarrow \) Henries/m
Conductivity \( \sigma \)  S/m \( \rightarrow \) Siemens/m

For lossless dielectrics, the permittivity is real and can be expressed as \( \varepsilon = \varepsilon_0 \varepsilon_r \), where \( \varepsilon_0 = 8.85 \times 10^{-12} \) F/m is the permittivity of free space, and \( \varepsilon_r \) is relative permittivity (dielectric constant)

For lossy dielectric materials, \( \varepsilon = \varepsilon_0 \varepsilon_r - j \varepsilon' \), where

\[
\varepsilon' = \frac{\sigma}{2\pi f}
\]
Reflection

For a good conductor \((f < \sigma/\varepsilon_r \varepsilon_0)\), terms \(\varepsilon_r\) and \(\sigma\) are generally insensitive to the operating frequency.

For lossy dielectrics, \(\sigma\) may be sensitive to frequency.

### Table 4.1 Material Parameters at Various Frequencies

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative Permittivity (\varepsilon_r)</th>
<th>Conductivity (\sigma) (s/m)</th>
<th>Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor Ground</td>
<td>4</td>
<td>0.001</td>
<td>100</td>
</tr>
<tr>
<td>Typical Ground</td>
<td>15</td>
<td>0.005</td>
<td>100</td>
</tr>
<tr>
<td>Good Ground</td>
<td>25</td>
<td>0.02</td>
<td>100</td>
</tr>
<tr>
<td>Sea Water</td>
<td>81</td>
<td>5.0</td>
<td>100</td>
</tr>
<tr>
<td>Fresh Water</td>
<td>81</td>
<td>0.001</td>
<td>100</td>
</tr>
<tr>
<td>Brick</td>
<td>4.44</td>
<td>0.001</td>
<td>4000</td>
</tr>
<tr>
<td>Limestone</td>
<td>7.51</td>
<td>0.028</td>
<td>4000</td>
</tr>
<tr>
<td>Glass, Corning 707</td>
<td>4</td>
<td>0.00000018</td>
<td>1</td>
</tr>
<tr>
<td>Glass, Corning 707</td>
<td>4</td>
<td>0.000027</td>
<td>100</td>
</tr>
<tr>
<td>Glass, Corning 707</td>
<td>4</td>
<td>0.005</td>
<td>10000</td>
</tr>
</tbody>
</table>
Reflection

When a radio wave propagating in one medium impinges upon another medium having different electric properties, this wave is partially reflected and partially transmitted.

If the second medium is a perfect conductor, then all incident energy is reflected back into the first medium without loss of energy.

Speed of propagation
\[ v_i = \frac{1}{\sqrt{\mu_i \varepsilon_i}} \]
(at free space 3x10^8 m/s)

Intrinsic impedance
\[ \eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}} \]
(at free space 120π Ω)
Reflection - Polarizations

(a) E-field in the plane of incidence  (b) E-field normal to the plane of incidence

(Vertical polarization)  (Horizontal polarization)
Reflection – Angle Relationships

Relationship between angles

$$\theta_i = \theta_r$$

Snell’s Law

$$\sqrt{\mu_1 \varepsilon_1 \sin(90 - \theta_i)} = \sqrt{\mu_2 \varepsilon_2 \sin(90 - \theta_t)}$$

When $\mu_1 = \mu_2$

$$\sqrt{\varepsilon_1 \sin(90 - \theta_i)} = \sqrt{\varepsilon_2 \sin(90 - \theta_t)}$$
Reflection – Magnitude Relationships

Reflection coefficient

\[ \frac{E_r}{E_i} = \Gamma \]

Transmission coefficient

\[ \frac{E_t}{E_i} = T = 1 + \Gamma \]

The value of \( \Gamma \) depends on polarization.

E-field in the plane of incident

\[ \Gamma_\parallel = \frac{E_r}{E_i} = \frac{\eta_2 \sin \theta_t - \eta_1 \sin \theta_i}{\eta_2 \sin \theta_t + \eta_1 \sin \theta_i} \]

E-field normal to the plane of incident

\[ \Gamma_\perp = \frac{E_r}{E_i} = \frac{\eta_2 \sin \theta_i - \eta_1 \sin \theta_t}{\eta_2 \sin \theta_i + \eta_1 \sin \theta_t} \]
Reflection – Magnitude Relationships

When the first medium is free space, and $\mu_1=\mu_2$

\[
\sin(90 - \theta_i) = \sqrt{\varepsilon_r} \sin(90 - \theta_t) \quad \sin \theta_t = \sqrt{1 - \cos^2 \theta_t} = \sqrt{1 - \frac{\cos^2 \theta_i}{\varepsilon_r}}
\]

\[
\Gamma_\parallel = \frac{E_r}{E_i} = \frac{\eta_2 \sin \theta_i - \eta_1 \sin \theta_i}{\eta_2 \sin \theta_i + \eta_1 \sin \theta_i} = \frac{\sin \theta_t - \sin \theta_i}{\sin \theta_t + \sqrt{\mu_0 \varepsilon_r \varepsilon_0} \sin \theta_i} = \frac{\sin \theta_t - \sqrt{\varepsilon_r} \sin \theta_i}{\sin \theta_t + \sqrt{\varepsilon_r} \sin \theta_i}
\]

\[
= \sqrt{1 - \frac{\cos^2 \theta_i}{\varepsilon_r} - \sqrt{\varepsilon_r} \sin \theta_i} = \frac{\varepsilon_r \sin \theta_i + \sqrt{\varepsilon_r - \cos^2 \theta_i}}{\varepsilon_r \sin \theta_i + \sqrt{\varepsilon_r - \cos^2 \theta_i}}
\]

\[
\Gamma_\perp = \frac{\sin \theta_i - \sqrt{\varepsilon_r - \cos^2 \theta_i}}{\sin \theta_i + \sqrt{\varepsilon_r - \cos^2 \theta_i}}
\]
Reflection – Magnitude Relationships

Parallel polarization (E-field in plane of incidence)

Perpendicular polarization (E-field not in the plane of incidence)
Reflection from Perfect Conductor

\[ \theta_i = \theta_r \]

Parallel / vertical polarization

\[ \theta_i = \theta_r \]
\[ E_i = E_r \]

Perpendicular / horizontal polarization

\[ \theta_i = \theta_r \]
\[ E_i = -E_r \]
Ground Reflection – Two-Ray Model

\[ E_{TOT} = E_{LOS} + E_g \]
Ground Reflection – Two-Ray Model
Ground Reflection – Two-Ray Model

General E-field expression in free space

\[ E(d,t) = \frac{E_0 d_0}{d} \exp\left( j \omega_c \left( t - \frac{d}{c} \right) \right) \]

where \( d_0 \) is a reference distance at the far-field.

Line-of-sight (LOS) wave

\[ E_{LOS}(d',t) = \frac{E_0 d_0}{d'} \exp\left( j \omega_c \left( t - \frac{d'}{c} \right) \right) \]

Ground reflection wave

\[ E_g(d'',t) = \Gamma \frac{E_0 d_0}{d''} \exp\left( j \omega_c \left( t - \frac{d''}{c} \right) \right) \]
Ground Reflection – Two-Ray Model

Assume horizontal E-field and perfect ground reflection, $\Gamma = -1$.

The combined E-field becomes

$$E_{\text{TOT}}(d, t) = E_{\text{LOS}}(d', t) + E_g(d'', t) = \frac{E_0 d_0}{d'} \exp \left( j \omega_c \left( t - \frac{d'}{c} \right) \right) - \frac{E_0 d_0}{d''} \exp \left( j \omega_c \left( t - \frac{d''}{c} \right) \right)$$

The path difference between the LOS and reflected paths is

$$\Delta = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

When $d \gg h_t + h_r$,

$$\sqrt{(h_t + h_r)^2 + d^2} = d \sqrt{1 + (h_t + h_r)^2 / d^2} \approx d + (h_t + h_r)^2 / (2d)$$

$$\sqrt{(h_t - h_r)^2 + d^2} = d \sqrt{1 + (h_t - h_r)^2 / d^2} \approx d + (h_t - h_r)^2 / (2d)$$

$$\Delta \approx \frac{(h_t + h_r)^2}{2d} - \frac{(h_t - h_r)^2}{2d} = \frac{2h_t h_r}{d}$$
Ground Reflection – Two-Ray Model

At some time, say at $t = \frac{d''}{c}$

$$E_{\text{TOT}}(d, t = \frac{d''}{c}) = \frac{E_0d_0}{d'} \exp\left(j \frac{\omega_c \Delta}{c}\right) - \frac{E_0d_0}{d''}$$

Also note that, when $d \gg h_t + h_r$,

$$\left| \frac{E_0d_0}{d'} \right| \approx \left| \frac{E_0d_0}{d''} \right| \approx \left| \frac{E_0d_0}{d} \right|$$

Denote $\theta_\Delta = \omega_c (\Delta / c)$, then

$$|E_{\text{TOT}}(d)| = \frac{E_0d_0}{d} \sqrt{(\cos \theta_\Delta - 1)^2 + \sin^2 \theta_\Delta}$$

$$= \frac{E_0d_0}{d} \sqrt{\cos^2 \theta_\Delta - 2 \cos \theta_\Delta + 1 + \sin^2 \theta_\Delta}$$

$$= \frac{E_0d_0}{d} \sqrt{2 - 2 \cos \theta_\Delta} = 2 \frac{E_0d_0}{d} \sin\left(\frac{\theta_\Delta}{2}\right)$$

Ref:

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta$$
Ground Reflection – Two-Ray Model

When $\theta_\Delta / 2$ is small ($\theta_\Delta / 2 = \omega_c (\Delta / 2c) < 0.3\text{rad}$)

$$\sin\left(\frac{\theta_\Delta}{2}\right) \approx \frac{\theta_\Delta}{2} = \frac{\omega_c \Delta}{2c} \approx \frac{\omega_c h_t h_r}{cd}$$

$$|E_{\text{TOT}}(d)| = 2 \frac{E_0 d_0}{d} \sin\left(\frac{\theta_\Delta}{2}\right) \approx 2 \frac{E_0 d_0}{d^2} \frac{2\pi h_t h_r}{\lambda} = \frac{k}{d^2}$$

Conclusion

For $d > \frac{20 h_t h_r}{\lambda}$

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

$$P_r(\text{dB}) = 40 \log d - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$$

It happens at:

$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$$

For example,

- $\lambda = 0.15\text{m}, h_t = 50\text{m}, h_r = 1.5\text{m}$
- $d = 10,000\text{m} = 10\text{km}$
Ground Reflection – Two-Ray Model

http://home.earthlink.net/~loganscott53/Two_Ray_Propagation.htm
Example 4.6
A mobile is located 5 km away from a base station and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E-field at 1 km from the transmitter is measured to be $10^{-3}$ V/m. The carrier frequency used for this system is 900 MHz.
(a) Find the length and the effective aperture of the receiving antenna.
(b) Find the received power at the mobile using the two-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5 m above ground.

Solution
Given:
T–R separation distance = 5 km
E-field at a distance of 1 km = $10^{-3}$ V/m
Frequency of operation, $f = 900$ MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333 \text{ m}.$$
(a) Length of the antenna, \( L = \frac{\lambda}{4} = \frac{0.333}{4} = 0.0833 \text{ m} = 8.33 \text{ cm} \).

Effective aperture of \( \frac{\lambda}{4} \) monopole antenna can be obtained using Equation (4.2).

Effective aperture of antenna = 0.016 m\(^2\).

(b) Since \( d \gg \sqrt{h_t h_r} \), the electric field is given by

\[
E_R(d) \approx \frac{2E_0 d_0 2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}
\]

\[
= \frac{2 \times 10^{-3} \times 1 \times 10^3}{5 \times 10^3} \left[ \frac{2\pi(50)(1.5)}{0.333(5 \times 10^3)} \right]
\]

\[
= 113.1 \times 10^{-6} \text{ V/m}.
\]

The received power at a distance \( d \) can be obtained using Equation (4.15)

\[
P_r(d) = \left( \frac{113.1 \times 10^{-6}}{377} \right)^2 \left[ \frac{1.8(0.333)}{4\pi} \right]^2
\]

\[
P_r(d = 5 \text{ km}) = 5.4 \times 10^{-13} \text{ W} = -122.68 \text{ dBW} \text{ or } -92.68 \text{ dBm}.
\]
When \( d_1, d_2 \gg h, h \gg \lambda \), the excess path length (difference between the direct path and the diffracted path) is

\[
\Delta \approx \frac{h^2}{2} \frac{d_1 + d_2}{d_1 d_2}
\]

The corresponding phase difference is

\[
\phi = \frac{2\pi \Delta}{\lambda} \approx \frac{2\pi}{\lambda} \frac{h^2}{2} \frac{d_1 + d_2}{d_1 d_2}
\]
Fresnel Zone Geometry

\[ r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} \]

is the radius corresponding to the \( n \)th Fresnel zone, which has \( n\lambda/2 \) path difference, or \( n\pi \) phase difference to the LOS.

A rule of thumb is that as long as 55% (many materials say 60%) of the first Fresnel zone is kept clear, the diffraction loss will be minimal.
Fresnel Zone Geometry

Fresnel Zone Boundary Calculator

A service of Green Bay Professional Packet Radio

Enter the Operating Frequency

Enter the Distance from Site A to the Obstruction

Enter the Distance from Site B to the Obstruction

N^{th} Fresnel Zone to Calculate

Submit Clear

http://gbppr.dyndns.org:8080/fresnel.main.cgi
Fresnel Zone Geometry

Observations:

\[ r_n = \sqrt{\frac{n \lambda d_1 d_2}{d_1 + d_2}} \]

- \( r_n \) is dependent of the wavelength (or frequency).
- If \( d_1 + d_2 \) is fixed, \( r_n \) takes smaller value when the position is closer to either end.
Knife-Edge Diffraction Geometry

- Diffraction allows radio signals to propagate around the curved surface or propagate behind obstructions.
- Based on Huygen’s principle of wave propagation.

(a) Knife-edge diffraction geometry. The point $T$ denotes the transmitter and $R$ denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.
(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if $\alpha$ and $\beta$ are small and $h \ll d_1$ and $d_2$, then $h$ and $h'$ are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.
(c) Equivalent knife-edge geometry where the smallest height (in this case $h_r$) is subtracted from all other heights.
Knife-Edge Diffraction Geometry

(a) $\alpha$ and $\nu$ are positive, since $h$ is positive

(b) $\alpha$ and $\nu$ are equal to zero, since $h$ is equal to zero

(c) $\alpha$ and $\nu$ are negative, since $h$ is negative
Knife-edge Diffraction Geometry

The field strength at point R is a vector sum of the fields due to all of the secondary Huygen’s sources in the plane above the knife edge.

Figure 4.13   Illustration of knife-edge diffraction geometry. The receiver R is located in the shadow region.
Knife-Edge Diffraction Geometry

Approximate diffraction gain expressions

\[
\nu = h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}}
\]

\[
G_d(dB) = 0 \quad \nu \leq -1 \quad (4.61.a)
\]

\[
G_d(dB) = 20\log(0.5 - 0.62\nu) \quad -1 \leq \nu \leq 0 \quad (4.61.b)
\]

\[
G_d(dB) = 20\log(0.5\exp(-0.95\nu)) \quad 0 \leq \nu \leq 1 \quad (4.61.c)
\]

\[
G_d(dB) = 20\log(0.4 - \sqrt{0.1184 - (0.38 - 0.1\nu)^2}) \quad 1 \leq \nu \leq 2.4 \quad (4.61.d)
\]

\[
G_d(dB) = 20\log\left(\frac{0.225}{\nu}\right) \quad \nu > 2.4 \quad (4.61.e)
\]
Knife-edge Diffraction Geometry

\[ v = h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} \]

Figure 4.14  Knife-edge diffraction gain as a function of Fresnel diffraction parameter \( v \).
Knife-edge Diffraction Geometry

- When there are multiple obstructions, the problem becomes much more complicated.
- A simple approach is to use a single equivalent obstacle.

![Diagram showing Knife-edge Diffraction Geometry](image)

**Figure 4.15** Bullington’s construction of an equivalent knife edge [from [Bul47] © IEEE].
Example 4.8
Given the following geometry, determine (a) the loss due to knife-edge diffraction, and (b) the height of the obstacle required to induce 6 dB diffraction loss. Assume $f = 900$ MHz.
Example

Solution

(a) The wavelength $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3} \text{ m.}$

Redraw the geometry by subtracting the height of the smallest structure.

\[\beta = \tan^{-1}\left(\frac{75 - 25}{10000}\right) = 0.2865^\circ\]

\[\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15^\circ\]
and
\[ \alpha = \beta + \gamma = 2.434^\circ = 0.0424 \text{ rad} \]

Then using Equation (4.56)
\[ \nu = 0.0424 \sqrt{\frac{2 \times 10000 \times 2000}{\sqrt{(1/3) \times (10000 + 2000)}}} = 4.24. \]

From Figure 4.14 or (4.61.e), the diffraction loss is 25.5 dB.

(b) For 6 dB diffraction loss, \( \nu = 0. \) The obstruction height \( h \) may be found using similar triangles \( (\beta = \gamma) \), as shown below.

It follows that \( \frac{h}{2000} = \frac{25}{12000} \), thus \( h = 4.16 \text{ m} \).
Scattering

- When a radio wave impinges on a rough surface, the reflected energy is spread out or diffused in all directions. (e.g., foliage).
- A surface is considered smooth if its minimum to maximum protuberance $h$ is less than the critical height.
  \[ h_c = \frac{\lambda}{8\sin \theta_i} \]
- The surface is considered rough if the protuberance is greater than $h_c$. 
• Modified reflection coefficient for rough surface

\[ \Gamma_{\text{rough}} = \rho_s \Gamma \]

• Scattering loss factor
  by Ament

\[ \rho_s = \exp \left[ -8 \left( \frac{\pi \sigma_h \sin \theta_i}{\lambda} \right)^2 \right] \]

modified by Boithias

\[ \rho_s = \exp \left[ -8 \left( \frac{\pi \sigma_h \sin \theta_i}{\lambda} \right)^2 \right] I_0 \left[ 8 \left( \frac{\pi \sigma_h \sin \theta_i}{\lambda} \right)^2 \right] \]

\( \sigma_h \): standard deviation of the surface height

\( I_0 \): Bessel function of the first kind and zero order
Scattering

Measured Reflection Coefficients of Rough Stone Wall

\( f = 4 \text{ GHz} \) \( \leftrightarrow \) Perpendicular Polarization (vertical antenna polarization)

Whittmann Hall location
Stone dielectric properties
\( \varepsilon_r = 7.51; \sigma = 0.028; \mu = 0.98 \)

Rough stone parameters
\( h = 12.7 \text{ cm}; \sigma_h = 2.54 \text{ cm} \)

- Ideal smooth surface
- Gaussian rough surface measured data
- Gaussian rough surface (w/ Bessel fit)

(a) E-field in the plane of incidence (parallel polarization).
Scattering

Measured Reflection Coefficients of Rough Stone Wall

\( f = 4 \text{ GHz} \) \( \rightarrow \) Parallel Polarization (horizontal antenna polarization)

- **Whittamore Hall location**
- **Stone dielectric properties**
  \( \varepsilon_r = 7.51; \sigma = 0.028; \mu = 0.95 \)
- **Rough stone parameters**
  \( h = 12.7 \text{ cm}; \sigma_h = 2.64 \text{ cm} \)

- Ideal smooth surface
- Gaussian rough surface
- Measured data
- Gaussian rough surface (w/ Bessel fcn)

(b) E-field normal to plane of incidence (perpendicular polarization).
Radar Cross Section (RCS) Model

\[
\text{RCS} = \frac{\text{Power density of scattered wave in direction of receiver}}{\text{Power density of radio wave incident on the scattering object}}
\]

For a medium and large size building located 5-10 km away, RCS values is in the range of 14.1 to 55.7 dBm².
Radar Cross Section (RCS) Model

\[ P_R = \frac{P_T G_T \lambda^2 \cdot \text{RCS}}{(4\pi)^3 d_T^2 d_R^2} \]

- \( P_T \) = Transmitted power (mW)
- \( G_T \) = Gain of transmitting antenna
- \( d_T \) = Distance of scattering object from transmitter
- \( d_R \) = Distance of scattering object from receiver

\[ P_R \text{(dBm)} = P_T \text{(dBm)} + G_T \text{(dBi)} + 20\log(\lambda) + \text{RCS}[\text{dB} \text{ m}^2] \]
\[ - 30\log(4\pi) - 20\log d_T - 20\log d_R \]
Practical Link Budget

• Most radio propagation models are derived using a combination of analytical and empirical models.

• Empirical approach is based on fitting curves or analytical expressions that recreate a set of measured data.

• Advantages:
  Takes into account all propagation factors, both known and unknown.

• Disadvantages:
  New models need to be measured for different environment or frequency.
Long-Distance Path Model

- Over many years, some classical propagation models have been developed, which are used to predict large-scale coverage for mobile communication system design.

Path loss at \(d_0 = P_T / P(d_0) = \overline{PL}(d_0)\)

Path loss at \(d = P_T / P(d) = \overline{PL}(d)\)

\[
\frac{\overline{PL}(d)}{\overline{PL}(d_0)} = \left(\frac{d}{d_0}\right)^n
\]

\[
\overline{PL}(d)[\text{dB}] = \overline{PL}(d_0)[\text{dB}] + 10n \log\left(\frac{d}{d_0}\right)
\]
# Long-Distance Path Model

## Table 4.2  Path Loss Exponents for Different Environments

<table>
<thead>
<tr>
<th>Environment</th>
<th>Path Loss Exponent, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free space</td>
<td>2</td>
</tr>
<tr>
<td>Urban area cellular radio</td>
<td>2.7 to 3.5</td>
</tr>
<tr>
<td>Shadowed urban cellular radio</td>
<td>3 to 5</td>
</tr>
<tr>
<td>In building line-of-sight</td>
<td>1.6 to 1.8</td>
</tr>
<tr>
<td>Obstructed in building</td>
<td>4 to 6</td>
</tr>
<tr>
<td>Obstructed in factories</td>
<td>2 to 3</td>
</tr>
</tbody>
</table>
Log-Normal Shadowing

• Long-distance path loss gives only the average value of path loss.
• Surrounding environment may be vastly different at two locations having the same T–R separation $d$.
• More accurate model includes a random variable to account for change in environment.

\[
PL(d)[\text{dB}] = \overline{PL}(d)[\text{dB}] + X_\sigma = \overline{PL}(d_0)[\text{dB}] + 10\log \left( \frac{d}{d_0} \right) + X_\sigma
\]

\[
P_r(d)[\text{dBm}] = P_t(d)[\text{dBm}] - PL(d)[\text{dB}]
\]

$X_\sigma$: Zero mean Gaussian random variable (dB)
$s_\sigma$: Standard deviation (dB)
Log-Normal Shadowing

![Graph showing signal strength vs distance]

- Signal strength (in dB) vs distance (in miles)
- Log-normal distribution indicated
- 8 dB marker at specific distances
- Normal distribution marker with deviation σ
Log-Normal Shadowing

- Values of $n$ and $\sigma$ are computed from measured data, using linear regression such that the difference between measured and estimated path losses is minimized in a mean square error sense over a wide range of measurement locations and T–R separations.

Figure 4.17 Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data, $n = 2.7$ and $\sigma = 11.8$ dB [from [Sei91] © IEEE].
Q Function

Q function or error function (erf) can be used to determine the probability that the received signal will exceed (or fall below) a particular level.

\[
Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{z}{\sqrt{2}}\right) \right]
\]

\[
Q(-z) = 1 - Q(z)
\]

\[
Q(0) = 0.5
\]

Some results of \(Q(z)\) and erf are listed in pages 647 and 649.

Matlab has \textit{erf} function.
Log-Normal Shadowing Model

The probability that the received signal level (in dB power unit) will exceed a certain value $\gamma$ can be calculated from the cumulative density function as

$$\Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - P_r(d)}{\sigma}\right)$$

The probability that the received signal level will be below $\gamma$ can be calculated from

$$\Pr[P_r(d) < \gamma] = Q\left(\frac{P_r(d) - \gamma}{\sigma}\right)$$
Gaussian Probability Density Function

\[ P(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left( -\frac{(x - m)^2}{2\sigma^2} \right) \]

\[ \Pr(x \geq x_0) \]
### Gaussian pdf-Q Function Relation

\[ P(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x-m)^2}{2\sigma^2} \right) \]

\[ P_r(x > x_0) = \int_{x_0}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x-m)^2}{2\sigma^2} \right) dx \]

Let \( y = \frac{x-m}{\sigma} \)

\[ P_r \left( x > \frac{x_0-m}{\sigma} \right) = \int_{\frac{x_0-m}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{y^2}{2} \right) dy = Q \left[ \frac{x_0-m}{\sigma} \right] = Q(z) \]

where \( z = \frac{x_0-m}{\sigma} \)
Given a circular coverage area of radius $R$...

There is a desired received signal level $\gamma$

We are interested in computing $U(\gamma)$, the percentage of useful service area (i.e., the percentage of area with a received power $P_R \geq \gamma$)

\[
U(\gamma) = \frac{1}{\pi R^2} \int Pr[P_r(r) < \gamma]dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R Pr[P_r(r) < \gamma]rdrd\theta
\]

\[
Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right) = \frac{1}{2} - \frac{1}{2} \text{erf}\left(\frac{\gamma - \overline{P_r(d)}}{\sigma\sqrt{2}}\right)
\]
Percentage of Coverage Area

\[ \Pr[P_r(d) > \gamma] = Q \left( \frac{\gamma - \overline{P_r(d)}}{\sigma} \right) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma - \overline{P_r(d)}}{\sigma \sqrt{2}} \right) \]

\[ = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma - [P_t - (\overline{PL}(d_0) + 10n \log(r / d_0))]}{\sigma \sqrt{2}} \right) \]

\[ = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{\gamma - [P_t - (\overline{PL}(d_0) + 10n \log(R / d_0) + 10n \log(r / R))]}{\sigma \sqrt{2}} \right) \]

Let \( a = \frac{\gamma - P_t + \overline{PL}(d_0) + 10n \log(R / d_0)}{\sigma \sqrt{2}} \), \( b = \frac{10n \log e}{\sigma \sqrt{2}} \)

\[ U(\gamma) = \frac{1}{2} - \frac{1}{R^2} \int_0^R r \cdot \text{erf} \left( a + b \ln \frac{r}{R} \right) dr \]

\[ = \frac{1}{2} \left\{ 1 - \text{erf}(a) + \exp \left( \frac{1 - 2ab}{b^2} \right) \left[ 1 - \text{erf} \left( \frac{1 - ab}{b} \right) \right] \right\} \]
Figure 4.18  Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.
Example

Example 4.9
Four received power measurements were taken at distances of 100 m, 200 m, 1 km, and 3 km from a transmitter. These measured values are given in the following table. It is assumed that the path loss for these measurements follows the model in Equation (4.69.a), where \( d_0 = 100 \) m:
(a) find the minimum mean square error (MMSE) estimate for the path loss exponent, \( n \); (b) calculate the standard deviation about the mean value; (c) estimate the received power at \( d = 2 \) km using the resulting model; (d) predict the likelihood that the received signal level at 2 km will be greater than -60 dBm; and (e) predict the percentage of area within a 2 km radius cell that receives signals greater than -60 dBm, given the result in (d).

<table>
<thead>
<tr>
<th>Distance from Transmitter</th>
<th>Received Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 m</td>
<td>0 dBm</td>
</tr>
<tr>
<td>200 m</td>
<td>-20 dBm</td>
</tr>
<tr>
<td>1000 m</td>
<td>-35 dBm</td>
</tr>
<tr>
<td>3000 m</td>
<td>-70 dBm</td>
</tr>
</tbody>
</table>
Solution

The MMSE estimate may be found using the following method. Let \( p_i \) be the received power at a distance \( d_i \) and let \( \hat{p}_i \) be the estimate for \( p_i \) using the \( (d/d_0)^n \) path loss model of Equation (4.67). The sum of squared errors between the measured and estimated values is given by

\[
J(n) = \sum_{i=1}^{k} (p_i - \hat{p}_i)^2
\]

The value of \( n \) which minimizes the mean square error can be obtained by equating the derivative of \( J(n) \) to zero, and then solving for \( n \).

(a) Using Equation (4.68), we find \( \hat{p}_i = p_i(d_0) - 10n \log(d_i + 100 \text{ m}) \). Recognizing that \( P(d_0) = 0 \text{ dBm} \), we find the following estimates for \( \hat{p}_i \) in dBm:

\[
\hat{p}_1 = 0, \quad \hat{p}_2 = -3n, \quad \hat{p}_3 = -10n, \quad \hat{p}_4 = -14.77n.
\]

The sum of squared errors is then given by

\[
\begin{align*}
J(n) &= (0 - 0)^2 + (-20 - (-3n))^2 + (-35 - (-10n))^2 + (-70 - (-14.77n))^2 \\
&= 6525 - 2887.8n + 327.153n^2
\end{align*}
\]

\[
\frac{dJ(n)}{dn} = 654.306n - 2887.8.
\]

Setting this equal to zero, the value of \( n \) is obtained as \( n = 4.4 \).
(b) The sample variance \( \sigma^2 = J(n)/4 \) at \( n = 4.4 \) can be obtained as follows.

\[
J(n) = (0 + 0) + (-20 + 13.2)^2 + (-35 + 44)^2 + (-70 + 64.988)^2 \\
= 152.36.
\]

\[
\sigma^2 = 152.36/4 = 38.09 \text{ dB}^2
\]

Therefore

\( \sigma = 6.17 \text{ dB} \), which is a biased estimate. In general, a greater number of measurements are needed to reduce \( \sigma^2 \).

(c) The estimate of the received power at \( d = 2 \text{ km} \) is given by

\[
\hat{p}(d = 2 \text{ km}) = 0 - 10(4.4)\log(2000/100) = -57.24 \text{ dBm}.
\]

A Gaussian random variable having zero mean and \( \sigma = 6.17 \text{ dB} \) could be added to this value to simulate random shadowing effects at \( d = 2 \text{ km} \).

(d) The probability that the received signal level will be greater than \(-60 \text{ dBm} \) is given by

\[
Pr[ P_r(d) > -60 \text{ dBm}] = Q\left(\frac{\gamma - \overline{P}_r(d)}{\sigma}\right) = Q\left(\frac{-60 + 57.24}{6.17}\right) = 67.4 \%.
\]

(e) If 67.4\% of the users on the boundary receive signals greater than \(-60 \text{ dBm} \), then Equation (4.78) or Figure 4.18 may be used to determine that 88\% of the cell area receives coverage above \(-60 \text{ dBm} \).
Outdoor Propagation Models

Radio transmission in a mobile communications system often takes place over irregular terrain.

The terrain profile of a particular area needs to be taken into account for estimating the path loss.

Most of the models are based on systematic interpretation of measurement data obtained in the service area.

Commonly used outdoor propagation models:

- Longley Rice model: point-to-point communication systems (40MHz–100MHz)
- Okumara model: widely used in urban areas (150 MHz – 300 MHz)
- Hata model: graphical path loss (150 MHz – 1500 MHz)
Outlines

Propagation Model - Attenuations
   - Free Space
      - Near-Field vs. Far-Field
   - Reflection
      - Two-Ray Model
   - Diffraction
      - Fresnel Zone
   - Scattering

Log-Normal Shadowing

Practical Link Budget Design
   - Outdoor Propagation Models