Chapter 7
Equalization, Diversity, and Channel Coding

- 1. Equalization -

Yimin Zhang, Ph.D.
Department of Electrical & Computer Engineering
Villanova University
http://yiminzhang.com/ECE8708
Outlines

• Equalization Techniques
• Algorithms for Adaptive Equalization

• Diversity Techniques
• RAKE Receiver

• Channel Coding
Introduction

• **Three techniques** are used independently or in tandem to improve receiver signal quality.

• **Equalization** compensates for ISI created by multipath with time dispersive channels \((\sigma_t > 0.1 \ T_s)\).

• **Diversity** also compensates for fading channel impairments, and is usually implemented by using two or more receiving antennas.

• The former counters the effects of time dispersion (ISI), while the latter reduces the depth and duration of the fades experienced by a receiver in a flat fading (narrowband) channel.

• **Channel Coding** improves mobile communication link performance by adding redundant data bits in the transmitted message.

• Channel coding is used by the Rx to detect or correct some (or all) of the errors introduced by the channel (Post detection technique).
Review: Frequency-Selective Fading

• If the channel possesses a constant-gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, then the channel creates frequency-selective fading.

\[
S(f) = \frac{1}{1 + G(f)H(f)}
\]

where \( S(f) \) is the signal spectrum, \( G(f) \) is the channel response, and \( H(f) \) is the transmitted signal spectrum. The bandwidth of the channel is \( B_c \), which is the bandwidth of the channel response.
Review: Frequency-Selective Fading

- Frequency-selective fading is due to time dispersion of the transmitted symbols within the channel.
  - Waveform is distorted by inter-symbol interference (ISI)
  - Equalization is required (Chapter 7)
- Frequency-selective fading channels are much more difficult to model than flat fading channels.
- For frequency-selective fading
  \[ B_S > B_C \]
  and
  \[ T_S < \sigma_\tau \]

More practically

\[ T_S < 10\sigma_\tau \quad \text{or} \quad \sigma_\tau > 0.1T_S \]
Equalization Techniques

Block diagram of a simplified communications system using an adaptive equalizer at the receiver.
Equalization Techniques

- Equalizer is usually implemented at baseband or at IF in a receiver.

- Baseband received signal

\[ y(t) = x(t) \ast f(t) + n_b(t) \]

- \( x(t) \): original information signal
- \( f(t) \): combined complex baseband impulse response of the transmitter, channel, and the RF/IF section of the receiver
- \( \ast \): convolution (the textbook used \( \otimes \))
- \( n_b(t) \): baseband noise at the input of the equalizer
Equalization Technologies

- Denote $h_{eq}(t)$ as the impulse response of the equalizer.

- The output of the equalizer is

$$d(t) = y(t) * h_{eq}(t) = x(t) * f(t) * h_{eq}(t) + n_b(t) * h_{eq}(t)$$

- If the impulse of the entire system (including the equalizer) satisfies

$$f(t) * h_{eq}(t) = \delta(t)$$

or, equivalently,

$$F(f)H_{eq}(f) = 1$$

Then, the ISI is cancelled.
Equalization – Frequency-Domain View

- If the channel is frequency selective, the equalizer enhances the frequency components with small amplitudes and attenuates the strong frequencies in the received frequency response.

\[ F(f) H_{eq}(f) = 1 \]
Equalization – Time-Domain View

- The time-domain impulse response of the channel / equalizer combination should be an impulse.

\[ f(t) \ast h_{eq}(t) = \delta(t) \]

- Symbol-based processing cannot achieve perfect equalization.
Equalization Techniques

• The term *equalization* can be used to describe any signal processing operation that minimizes ISI.

• For a time-varying channel, an adaptive equalizer is needed to track the channel variations.

• Two operation modes for an adaptive equalizer: training and tracking.

• Three factors affect the time spanning over which an equalizer converges: equalizer algorithm, equalizer structure and time rate of change of the multipath radio channel.

• TDMA wireless systems are particularly well suited for equalizers.
Basic Structure of Adaptive Equalizer

- Transversal filter with $N$ delay elements, $N+1$ taps, and $N+1$ tunable complex weights
- These weights are updated continuously by an adaptive algorithm
- The adaptive algorithm is controlled by the error signal $e_k$

A basic linear equalizer during training.
Equalization Techniques

- Classical equalization theory: using training sequence (non-blind) to minimize the cost function

\[ E[|e_k(t)|^2] = E[e_k(t)e_k^*(t)] \]

- Recent techniques for adaptive algorithm: blind algorithms
  - Constant Modulus Algorithm (CMA, used for constant envelope modulation)
  - Spectral Coherence Restoral Algorithm (SCORE, exploits spectral redundancy or cyclostationarity in the Tx signal)
Solutions for Optimum Weights

Note: we used $w^*$ as the weights in the formulation.

- Error signal
  \[ e_k = x_k - w^H_k y_k \]
  where
  \[ y_k = [y_k, y_{k-1}, y_{k-2}, \ldots, y_{k-N}]^T \]
  \[ w_k = [w_k, w_{k-1}, w_{k-2}, \ldots, w_{k-N}]^T \]

- Mean square error
  \[ |e_k|^2 = |x_k|^2 + w_k^H y_k y_k^H w_k - x_k w_k^T y_k^* - x_k^* w_k^H y_k \]
Solutions for Optimum Weights

- **Mean square error (MSE)**
  \[
  |e_k|^2 = |x_k|^2 + \mathbf{w}_k^H \mathbf{y}_k \mathbf{y}_k^H \mathbf{w}_k - x_k \mathbf{w}_k^T \mathbf{y}_k^* - x_k^* \mathbf{w}_k^H \mathbf{y}_k
  \]

- **Expected MSE**
  \[
  \xi = E[|e_k|^2] = E[|x_k|^2] + \mathbf{w}_k^H \mathbf{R} \mathbf{w} - \mathbf{w}_k^H \mathbf{p} - \mathbf{w}_k^T \mathbf{p}^*
  \]

where

\[
\mathbf{R} = E[\mathbf{y}_k^H \mathbf{y}_k] = E\left[
\begin{bmatrix}
|y_k|^2 & y_k y_{k-1}^* & y_k y_{k-2}^* & \cdots & y_k y_{k-N}^* \\
y_{k-1} y_k^* & |y_{k-1}|^2 & y_{k-1} y_{k-2}^* & \cdots & y_{k-1} y_{k-N}^* \\
y_{k-2} y_k^* & y_{k-2} y_{k-1}^* & |y_{k-2}|^2 & \cdots & y_{k-2} y_{k-N}^* \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
y_{k-N} y_k^* & y_{k-N} y_{k-1}^* & y_{k-N} y_{k-2}^* & \cdots & |y_{k-N}|^2
\end{bmatrix}
\right]
\]

**Input correlation matrix**

**Or input covariance matrix**

\[
\mathbf{p} = E[\mathbf{x}_k^* \mathbf{y}_k] = E\left[
\begin{bmatrix}
x_k^* y_k & x_k^* y_{k-1} & x_k^* y_{k-2} & \cdots & x_k^* y_{k-N}
\end{bmatrix}
\right]^T
\]
Solutions for Optimum Weights

- Optimum weight vector is obtained from

\[ \nabla = \frac{\partial \xi}{\partial \mathbf{w}} = 2\mathbf{R}\mathbf{w} - 2\mathbf{p} = 0 \]

as

\[ \hat{\mathbf{w}} = \mathbf{R}^{-1}\mathbf{p} \]

(provided that \( \mathbf{R} \) is not singular, or \( \mathbf{R}^{-1} \) exists)

- Minimum mean square error (MMSE)

\[ \xi_{\text{min}} = E[|x_k|^2] - \mathbf{p}^H \mathbf{R}^{-1} \mathbf{p} = E[|x_k|^2] - \mathbf{p}^H \hat{\mathbf{w}} \]

- Minimizing the MSE tends to reduce the bit error rate.

- Note that, the exact statistical expectation cannot be obtained in practice.
Solutions for Optimum Weights
Equalization Techniques

- Two general categories - linear and nonlinear equalization.
- If the equalizer output is not used in the feedback path to adapt the equalizer, the equalization is linear.
- If the equalizer output is fed back to change the subsequent outputs of the equalizer, the equalization is nonlinear.
Equalizer Techniques

- Linear transversal equalizer (LTE, made up of tapped delay lines)

\[
y(t) + n_b(t) \xrightarrow{Ts} \xrightarrow{Ts} \xrightarrow{Ts} \xrightarrow{Ts}
\]

delay elements

clock with period \(\tau\)

Basic linear transversal equalizer structure
Structure of a Linear Transversal Equalizer

\[ \hat{d}_k = \sum_{n=-N_1}^{N_2} C_n^* y_{k-n} \]

- \( E[|e(n)|^2] = \frac{T}{2\pi} \int_{-T}^{T} \left| \frac{N_o}{F(e^{j\omega})^2 + N_o} \right| d\omega \)

- \( F(e^{j\omega}) \) : frequency response of the channel
- \( N_o \) : noise spectral density
Structure of a Lattice Equalizer
Equalizer Techniques

Infinite impulse response (IIR) filter - Not often used because of the stability concerns.
Characteristics of Lattice Filter

- **Advantages**
  - Numerical stability
  - Faster convergence
  - Unique structure allows the dynamic assignment of the most effective length

- **Disadvantages**
  - The structure is more complicated
Nonlinear Equalization

• Used in applications where the channel distortion is too severe

• Three effective methods
  – Decision Feedback Equalization (DFE)
  – Maximum Likelihood Symbol Detection
  – Maximum Likelihood Sequence Estimator (MLSE)
Nonlinear Equalization--DFE

- Basic idea: once an information symbol has been detected and decided upon, the ISI that it induces on future symbols can be estimated and substracted out before detection of subsequent symbols.

- Nonlinearity comes from the fact that the decision output cannot be expressed as a linear function of the decision input (which is a linear combination of the input signals).

- Can be realized in either the direct transversal form or as a lattice filter.
Nonlinear Equalizer-DFE

\[ \hat{d}_k = \sum_{n=-N_1}^{N_2} C_n^* y_{k-n} + \sum_{i=1}^{N_3} F_i^* d_{k-i} \]

\[ E\left[|e(n)|^2\right]_{\text{min}} = \exp\left\{ \frac{T}{2\pi} \int_{-\pi}^{\pi} \ln\left[ \frac{N_0}{|F(e^{j\omega})|^2 + N_0} \right] d\omega \right\} \]

- The MMSE for the DFE is always smaller than that of the LTE, particularly when there are nulls in \(|F(e^{j\omega})|\).
Nonlinear Equalization--MLSE

- MLSE tests all possible data sequences (rather than decoding each received symbol by itself), and chooses the data sequence with the maximum probability as the output.
- Usually has a large computational requirement.
Nonlinear Equalizer-MLSE

Fig. 10 The structure of a maximum likelihood sequence equalizer (MLSE) with an adaptive matched filter

- MLSE requires knowledge of the channel characteristics in order to compute the matrices for making decisions
- MLSE also requires knowledge of the statistical distribution of the noise corrupting the signal
Algorithm for Adaptive Equalization

- Performance measures for an algorithm
  - Rate of convergence
  - Misadjustment
  - Computational complexity
  - Numerical properties

- Factors dominate the choice of an equalization structure and its algorithm
  - The cost of computing platform
  - The power budget
  - The radio propagation characteristics
Algorithm for Adaptive Equalization

- The speed of the mobile unit determines the channel fading rate and the Doppler spread, which is related to the coherent time of the channel directly.

- The choice of algorithm, and its corresponding rate of convergence, depends on the channel data rate and coherent time.

- The number of taps used in the equalizer design depends on the maximum expected time delay spread of the channel.

- The circuit complexity and processing time increases with the number of taps and delay elements.
Algorithm for Adaptive Equalization

- Three classic non-blind equalization algorithms:
  - zero forcing (ZF),
  - sample matrix inversion (SMI),
  - least mean squares (LMS), and
  - recursive least squares (RLS) algorithms

- Two commonly used blind equalization algorithms (blind means the algorithms do not require training or pilot signals, but they use some specific properties of the signals)
  - Constant Modulus Algorithm (CMA, used for constant envelope modulation)
  - Spectral Coherence Restoral Algorithm (SCORE, exploits spectral redundancy or cyclostationarity in the Tx signal)
SMI uses data samples to estimate $\mathbf{R}$ and $\mathbf{p}$, and the optimum weight vector is approximated as

$$\hat{\mathbf{w}}_{\text{SMI}} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{p}}$$

How to estimate $\mathbf{R}$ and $\mathbf{p}$?

$$\hat{\mathbf{R}}(t) = \beta \hat{\mathbf{R}}(t-1) + (1 - \beta)\mathbf{y}(t)\mathbf{y}^H(t) \quad \hat{\mathbf{R}}(1) = \mathbf{y}(1)\mathbf{y}^H(1)$$

$$\hat{\mathbf{p}}(t) = \beta \hat{\mathbf{p}}(t-1) + (1 - \beta)\mathbf{y}(t)x^*(t) \quad \hat{\mathbf{p}}(1) = \mathbf{y}(1)x^*(1)$$

where $0<\beta<1$ is a forgetting factor.

The following matrix inversion formula is often used in practice to avoid large-scale matrix inversion

$$\hat{\mathbf{R}}^{-1}(t) = \beta^{-1} \hat{\mathbf{R}}^{-1}(t-1) - \frac{\beta^{-2} \hat{\mathbf{R}}(t-1)\mathbf{y}(t)\mathbf{y}^H(t)\hat{\mathbf{R}}(t-1)}{1 + \beta^{-1} \mathbf{y}^H(t)\hat{\mathbf{R}}(t-1)\mathbf{y}(t)}$$
Implementations - LMS

The steepest descent method updates the weight vector by

\[ w(t + 1) = w(t) - \frac{\mu}{2} \nabla_w E \left[ |e(t)|^2 \right] \]

where \( \mu \) is the step size, and

\[ \nabla_w E \left[ |e(t)|^2 \right] = -2p + 2Rw(t) \]

\[ R = E[yy^H(t)], \quad p = E[y(t)x^*(t)] \]

LMS uses instantaneous values to approximate statistical average,

\[ \hat{R} = y(t)y^H(t), \quad \hat{p} = y(t)x^*(t) \]

\[ \nabla_w E \left[ |e(t)|^2 \right] \approx -2y(t)x^*(t) + 2y(t)y^H(t)w(t) = -2y(t)e^*(t) \]
Implementations - LMS

Therefore, LMS updates the weight vector by

\[ w(t + 1) = w(t) + \mu y(t)e^*(t) \]

To ensure convergence, \( \mu \) should be chosen such that

\[ 0 < \mu < \frac{1}{\lambda_{\text{max}}} \]

where \( \lambda_{\text{max}} \) is the maximum eigenvalue of \( R \). In practice, the following condition is often used instead

\[ 0 < \mu < \frac{1}{\text{trace}(R)} = \frac{1}{\text{total input power}} \]
Implementations - RLS

LMS is simple, but it converges slowly.

RLS improves the convergence by using

\[ w(t + 1) = w(t) - \gamma R^{-1}(t)y(t + 1)e^*(t + 1) \]

where

\[
\begin{align*}
R(0) &= \delta I \\
R(t) &= \alpha R(t - 1) + y(t)y^H(t)
\end{align*}
\]

\[ e(t + 1) = r(t) - w^H(t)y(t + 1) \]

\[ \gamma = \frac{1}{\alpha + y^H(t + 1)R^{-1}(t)y(t + 1)} \]

\( \delta \) is a small positive number.

The inversion of \( R(t) \) is done recursively.
Implementations – Blind Methods

Although non-blind equalizers provide better performance, the use of training or pilot signals reduces bandwidth efficiency and, therefore, is often undesired.

Also, the estimation of channels is often difficult, particularly in fading environment.

Blind equalization methods refer to those without the information of the training signal or channel information.

However, some kind of information/assumption is required. The following are those often used in practice:

- finite alphabets
- constant modulus
When the desired signals take discrete values, the receiver output is close to the desired signal when the channel distortion is not severe.

When the demodulation output is used as a reference, it is called decision-directed (DD) method.
Implementations – DD

Input Signal

\[ y_k \xrightarrow{\mathbf{z}^{-1}} y_{k-1} \xrightarrow{\mathbf{z}^{-1}} y_{k-2} \cdots \xrightarrow{\mathbf{z}^{-1}} y_{k-N} \]

\[ w_{0k} \xrightarrow{\mathbf{z}^{-1}} w_{1k} \xrightarrow{\mathbf{z}^{-1}} w_{2k} \cdots \xrightarrow{\mathbf{z}^{-1}} w_{Nk} \]

Adaptive Algorithm that updates each weight \( w_{nk} \)

A basic linear equalizer during training.

\[ \hat{d}_k \text{ output of equalizer} \]

\[ d_k \text{ is set to } x_k \text{ or represents a known property of the transmitted signal} \]
The object of the CMA is to restore the equalizer output to have a constant instantaneous modulus. This can be done by choosing the coefficient vector \( \mathbf{w} \) in such a way as to minimize a positive definition function of the signal modulus variation.

Assume

\[
|x(t)| = 1
\]

Define cost function as

\[
J = \frac{1}{4} E \left\{ \left[ |z(t)|^2 - 1 \right]^2 \right\}
\]

where

\[
z(t) = \mathbf{w}^H (t) \mathbf{y}(t)
\]

is the equalizer output.
Implementations – CMA

The weight vector is updated according to

\[
\mathbf{w}(t + 1) = \mathbf{w}(t) - \mu \nabla_\mathbf{w} J(t)
\]

\[
J(t) = \frac{1}{4} \left[ |z(t)|^2 - 1 \right]^2
\]

where \( \mu \) is an adaptation constant.

As the result, we have

\[
\mathbf{w}(t + 1) = \mathbf{w}(t) - \mu \{ |z(t)|^2 - 1 \} z(t)^* y(t)
\]

Let

\[
\tilde{\varepsilon}(t) = [ |z(t)|^2 - 1 ] z(t)
\]

then

\[
\mathbf{w}(t + 1) = \mathbf{w}(t) - \tilde{\varepsilon}^* (t) y(t)
\]
Implementations – CMA

Properties of CMA

1) Shape of the performance function $J$

$$J = \frac{1}{4} E\{[|z(t)|^2 - 1]^2\}$$

$J$ is minimized with a value of zero at any point on a circle with unit radius and centered on the origin.

2) Phase Roll

$$w = e^{j\phi} w_{opt}, \quad z(t) \rightarrow e^{j\phi} z(t)$$

The phase shift to the equalizer is not uniquely determined.

For phase-modulation signals, the phase roll must be estimated and removed, or use differential modulation where the absolute value of the phase is not important.
Outlines

• **Equalization Techniques**
  – To reduce …

• **Types and Algorithms for Adaptive Equalization**
  – Linear vs. nonlinear
  – Blind vs. non-blind

• **Most commonly used algorithms**
  – LMS/RLS : linear non-blind
  – DFE/MLSE : nonlinear non-blind
  – DD/CMA/SCORE : blind